TABLE 1. Dependence of Heat-Transfer Coefficient α [kcal/(m^2·h·deg)] on Flow Velocity v (m/sec)

v	100	150	200	240	290	330	360	400	550
α	350	430	490	560	590	630	680	710	820

The data obtained agree well with the empirical dependence $\alpha \sim v^{0.6}$ found earlier [7].

NOTATION

α, heat-transfer coefficient; c_V, heat capacity per unit volume of specimen material; β, coefficient of thermal expansion; t, time; l, length of cylindrical specimen; R, radius of cylinder; q, total power of heat losses; T(x), temperature field along specimen; I, current strength; U, voltage drop over specimen; Δl, limiting absolute elongation of specimen.

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COMPRESSION MECHANISM FOR TWO-COMPONENT LOOSE MEDIA MODELED BY SPHERICAL PARTICLES

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The packing density coefficient of two-component granular media, modeled by steel spheres, is investigated for a change in the fractional and concentration compositions of the mixture in a broad range of ratios between the diameters of the particles being mixed.

The mixing of bodies differing sharply in size results in a perceptible rise in the packing density coefficient (K) of a loose mass. In fact, the minimal porosity ($\Pi = 1 - K$) of large volumes filled by irregular compacted identical spherical particles is independent of the particle size and equals $\Pi_0 \approx 0.36$ [1]. The minimum porosity of a binary loose mixture with sharply differing component sizes is achieved by filling all the pores between the large spheres with fine fractions. Hence, the latter occupy 64% of the total volume

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of the void and the porosity of the aggregate will diminish in the limit ($d_2 >> d_1$) to $\Pi = 0.36 \cdot 0.36 \approx 0.13$ for a content of $\approx$26 vol.% of the fine fraction in the mixture.

The mechanism of the change in the coefficient K of two-component mixtures with approximately identical particle sizes $d_2/d_1 < 2$ is not evident when an additional particle cannot be placed in the pores between the large spheres. In particular, for small d_2/d_1 an additive change in the mixture volume is often postulated [2]. This means that compression of the volume $\omega = 0$, and since $\omega = (K - K_0)/K$, the mixture packing coefficient K should not vary: $K = K_0$.

The impression is produced that compression occurs in two-component mixtures only when fine particles are placed in the pores between the large spheres.

The behavior of two-component granular media is investigated in this paper for a change in the fractional and concentration compositions in order to clarify the mechanism of their compression in a broad interval of d_2/d_1. In contrast to the known papers [3] in which an analogous problem was solved for individual compositions of granular media by not strictly calibrated fractions, we systematically investigated mixtures whose components were identical steel spheres within 0.2% limits for particles of nonspherical configuration. Only in such a formulation of the question could a solution of theoretical value be obtained.

The characteristics obtained are of interest for the structural theory of granular media [3]. Questions of the packing of irregular structure are simultaneously in direct connection with problems of the theory of solutions which uses these results to produce models of the liquid state [4].

Steel balls of diameter d = 0.8, 1, 1.26, 2, 2.51, 3, 3.95, and 5.13 mm were used in the experiments. To diminish near-wall effects, the method of a reservoir with flexible walls [5] was used. These walls were in the form of thin rubber tanks which were filled by a mixture of balls of two sizes in the concentration chosen. The air was evacuated from the tank with the mixture, and the aggregate obtained, which had a spherical shape, was subjected to multilateral compression to obtain an irregularly compactly packed statistical set of particles. The mixture was first mixed carefully for a uniform particle distribution. This is easily achieved for mixtures with a low d_2/d_1 ratio. For large d_2/d_1 when small balls slip into the pores between the large spheres, especially for a high concentration of these latter, it is difficult to obtain a uniform particle distribution. However, the clustering of small balls in separate parts of the mixture does not influence the final result, which was verified by investigating a mixture of constant composition after multiple mixing. The spread in K did not exceed 0.5% and was within the limits of measurement accuracy.

The space occupied by the balls (v_0) was determined by suspending the tank with the mixture in air and in alcohol with the density of the ball material taken into account. The intrinsic volume of the balls and the packing density $K = v_s/v_0$ were calculated for each composition. Dependences $K = f(v_i)$ were consequently constructed for different ratios d_2/d_1, where v_i is the volume fraction of balls of one size. These dependences are shown in Fig. 1. They are constructed under the conditions of adding coarse balls to the finer fraction. Hence, the initial one-component mass contained not less than $3 \cdot 10^4$ balls and had a $K_0 = 0.646$ packing density coefficient, which agrees well with the known value of $K_0 = 0.64$, determined for an infinite volume of a single-component loose mass [1, 5].

As follows from the experimental results (Fig. 1), mixing unequal balls always results in compression. The maximum value of the packing coefficient (K_{max}) increases with the increase in the ratio between the diameters of the pairs being mixed and apparently approaches the value K = 0.87 asymptotically as $d_2/d_1 \to \infty$ (Fig. 2).

In connection with the facts noted, the following compression mechanism for two-component mixtures can be assumed. Replacing the small spheres by large ones results in compression, since the space occupied by a definite quantity of small spheres and the void between them turns out to be occupied by the large spheres. At the same time the large spheres produce local distortions of the initial structure of the fine fractions, forming a new kind of void in contact with the small balls. Hence, the introduction of the large spheres with a total volume V_2 in the initial one-component mass of fine fractions increases the volume it occupies V_1/K_0 by the quantity $V_2 + \alpha$, where α is the total volume of the new kind of void. Therefore, the packing density coefficient for small volume contents of the coarse fraction can be represented for two-component mixtures by the formula

$$K = \frac{V_1 + V_2}{\frac{V_1}{K_0} + V_2 + \alpha}. \quad (1)$$

The volume of single pores in the contact between the large sphere with the fine balls increases as d_2/d_1 increases and becomes a maximum at the plane of the wall. However, the total surface grows in inverse proportion to the diameter of the dispersed particles upon division of a spherical volume by spherical particles, and therefore their total volume will be significantly less for larger values of d_2/d_1 and the same volume composition of the coarse fraction.

The following experiments were performed to estimate the influence of the new type of voids near the spheres on the packing process: A large sphere and the maximum quantity of small spheres in one layer packed on it were covered by a rubber tank. The quantity $\lambda = (v_0 - v_s)/v_s$ was calculated, where v_0 is, as before, the volume of the whole aggregate determined by suspending the tank with the spheres in air and in alcohol, and v_s is the total volume of the spheres. The dependence of the maximum quantity of fine spheres n making simultaneous contact with a sphere of diameter d_2 is shown in Fig. 2. These results are described satisfactorily by the inequality

$$\pi\left(\frac{d_2}{d_1}+1\right)^2 \leqslant n \leqslant \frac{2\sqrt{3}}{3}\pi\left(\frac{d_2}{d_1}+1\right)^2,$$

which we present without proof.

It should be noted that such an estimate of n agrees with the results in [6], in which n was determined more rigorously up to $d_2/d_1 = 2.0$.

As follows from the results in Fig. 2, the quantity $\lambda = (v_0 - v_s)/v_s = (1/K) - 1$ increases as d_2/d_1 diminishes and $(1/K_0) - 1 = 0.26/0.74 = 0.35$ for $d_2 = d_1$ when n = 12 (Fig. 2), which characterizes the densest "regular" arrangement of particles (K = 0.74). Hence, the experimentally determined dependence $\lambda = f(d_2/d_1)$ (Fig. 2) is only a rough approximation of the situation observed in mixtures, since the maximum quantity of fine spheres is stacked on the surface of the sphere in this case. The number of spheres making contact n' in a statistical mixture will be much less and $n' \approx 8$ [7], for example, for a one-component mixture when $K_0 = 0.64$.

In fact, a diminution in the ratio d_2/d_1 [1] hinders a regular particle arrangement in the mixture, and the structure of the mixture becomes completely irregular as $d_2/d_1 \to 1$. Hence, the curve $\lambda = (v_0 - v_s)/v_s$ should result in the value $(1/K_0) - 1 = 0.354/0.646 = 0.55$ for irregularly compactly stacked particles with $d_2 = d_1$.

In order to clarify the nature of the change in the inner particle configuration by a method analogous to [7], the coordination number n' of the mixture components was investigated for different d_2/d_1. An acetone dye was introduced in a mixture of spheres containing ≈1 numerical percent of large spheres, and after it had dried the loose material was converted into a hard aggregate (skeleton) permitting the computation of n' by means of tracks of the contacts.

The results of these calculations (Fig. 2) show that for small d_2/d_1 a significantly lower quantity of small spheres make contact with the large spheres than for a regular arrangement. It can be noted that the relative difference in the coordination numbers $\Delta n/n$ of regular and irregular structures diminishes monotonically and practically vanishes for $d_2/d_1 > 3$. This indicates that the complete order of the particles observed on flat walls $d_2 \gg d_1$ goes smoothly over into an irregular arrangement with the diminution in d_2/d_1 and results in complete disorder for $d_2 = d_1$.

The increment in the volume of voids near the spheres induced by unit volume of the sphere $(v_0 - v_s)/v_s$ grows for fixed d_2/d_1 when going from the regular to the irregular particle arrangement, because of the diminution in the coordination number. In a first approximation it can be assumed that the relative increment in the volume of the voids $\Delta\lambda/\lambda = (\lambda' - \lambda)/\lambda$ is proportional to the relative diminution in the coordination number $\Delta n/n = (n - n')/n$. Taking into account that $\Delta n/n = 0.333$ causes a relative increase in the voids $\Delta\lambda/\lambda = 0.572$ for a one-component system, $\Delta\lambda$ can be calculated by knowing the difference in the coordination numbers Δn, from the formula

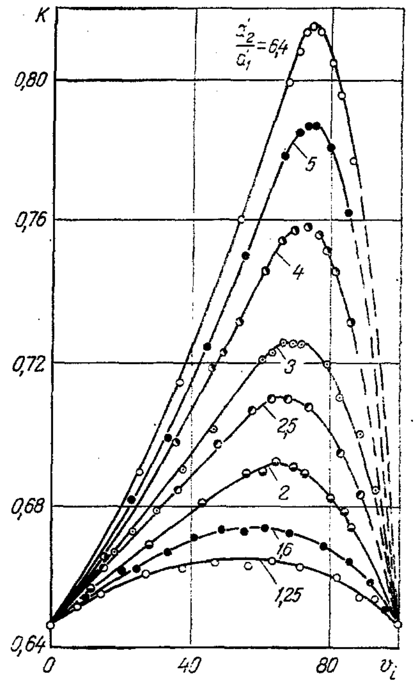

Fig. 1

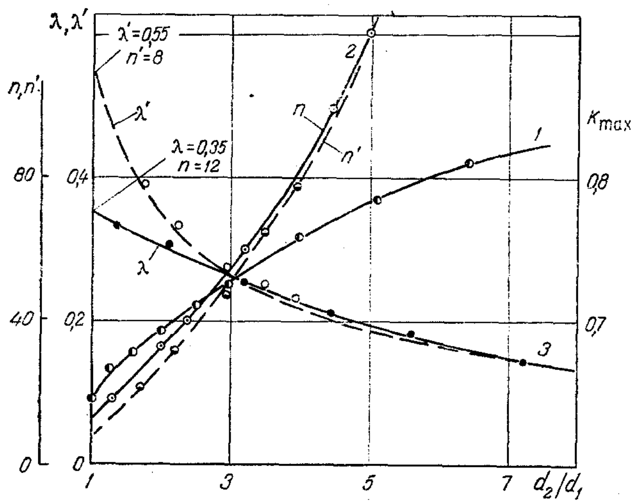

Fig. 2

Fig. 1. Concentration dependence of the packing density coefficient of two-component mixtures with a different ratio between the diameters of the pairs being mixed; v_i is the volume percent of large balls.

Fig. 2. Structural characteristics of two-component mixtures: 1) dependence of the maximum packing density coefficient (K_{max}) on the ratio between the ball diameters; 2) number of balls "regularly" (n) and irregularly (n') making contact with a ball of diameter $d_2 > d_1$; 3) change in the quantity $\lambda = (v_0 - v_s)/v_s$ for the "regular" λ and irregular λ' (open circles) locations of fine balls on a sphere.

$$\Delta\lambda = 1.72\,\frac{\lambda\cdot\Delta n}{n}. \tag{2}$$

The results of these computations as well as the approximating curve

$$\lambda' = \left(\frac{1}{K_0} - 1\right)\left(\frac{d_1}{d_2}\right)^{2/3} \tag{3}$$

are presented in Fig. 2.

The dependence (3) permits computation of the volume of the new type of void per unit volume of an irregularly densely packed system of spheres for a given ratio between their diameters. The total volume of voids is

$$\alpha = V_2\left(\frac{1}{K_0} - 1\right)\left(\frac{d_1}{d_2}\right)^{2/3} \tag{4}$$

for a volume V_2 of spheres of the coarse fraction contained in the mixture.

It can be noted that the quantity α determined in this manner satisfies the boundary conditions

$$\frac{d_1}{d_2} = 1;\quad \alpha = \left(\frac{1}{K_0} - 1\right)V_2;\quad K = K_0;$$

$$\frac{d_1}{d_2} \to 0;\quad \alpha \to 0;\quad V_1 \to 0.26;\quad K \to 0.87;$$

and (1), taking account of (4), describes the ascending part of the curve $K = f(v)$ in the region of the maximum to an accuracy not lower than 1.5%. A "skeleton" of coarse fractions is formed in the region of the maximum and a further rise in v_2 is accompanied by the origination of additional voids between the large spheres, which results in a diminution in K. Hence, the compression mechanism described by (4) is valid to $K = K_{max}$.

Therefore, two factors play the main role in the mixing of large and fine spheres. The first factor is related to the selection of the existing voids between the small spheres upon the addition of coarse spheres and always results in compression. The second decompressing factor is related to the formations of the new kind of voids in the contacts between the small and large spheres. The compression factor is simultaneously always predominant over this latter, and hence the mixing of arbitrarily small spherical bodies of different size results in a rise in the packing density of the mixture.

NOTATION

K_0, K, packing density coefficients of a one- and two-component loose medium, respectively; Π_0, Π, their porosity; ω, compression of the volume; d_1, d_2, the diameters of the particles being mixed ($d_2 > d_1$); v_s, intrinsic volume of the spheres; v_0, the volume they occupy; v_i, the volume fraction of the i-th component; V_1, V_2, volumes of the fractions; n, λ, coordination numbers of the large sphere and near-sphere void for a "regular" particle arrangement in the mixture; n', λ', the same, for an irregular structure; α, total volume of near-spherical voids in the mixture.

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